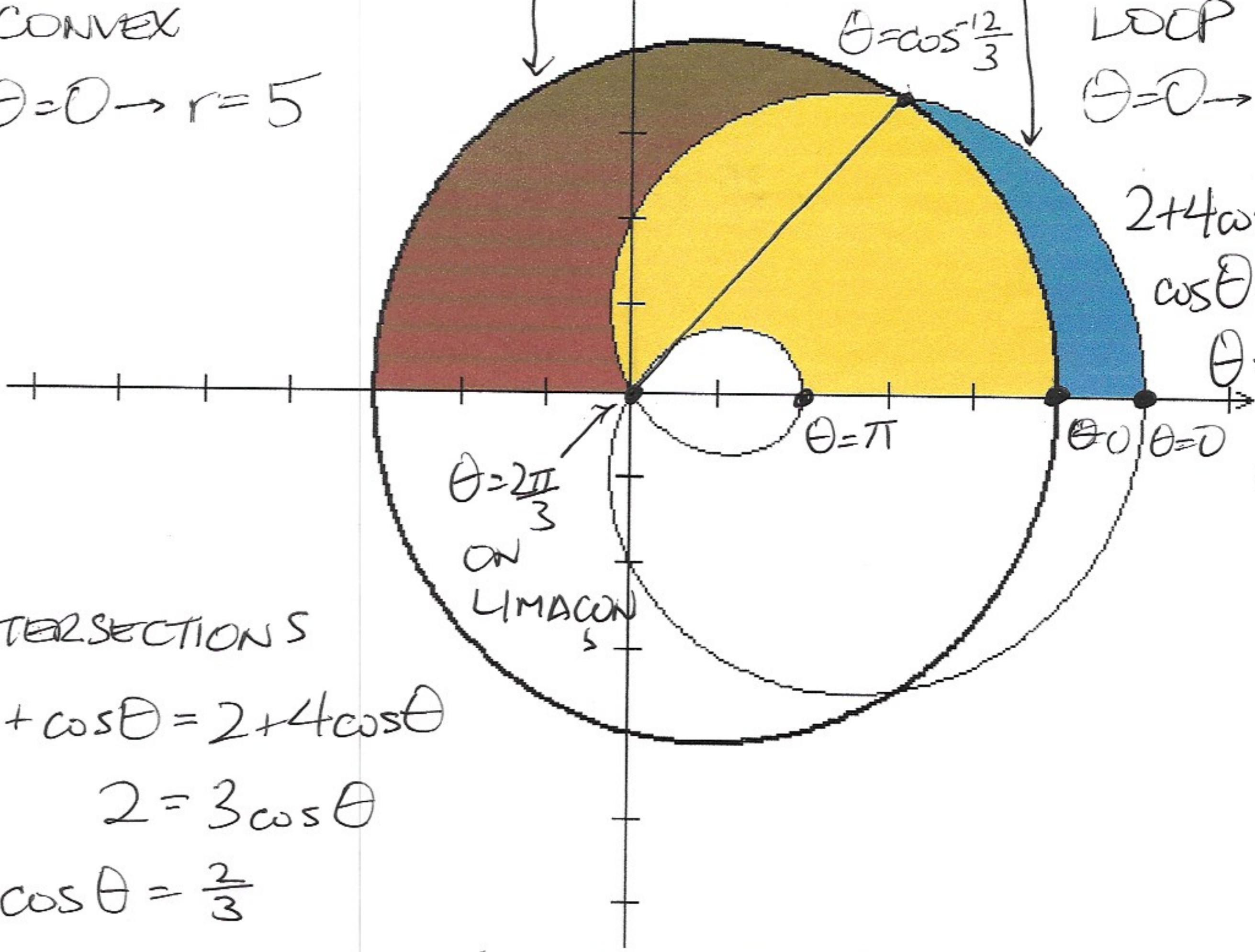
CONVEX

$\theta = 0 \rightarrow r = 5$

LOOP

$\theta = 0 \rightarrow r = 6$



$2 + 4\cos\theta = 0$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

INTERSECTIONS

$4 + \cos\theta = 2 + 4\cos\theta$

$2 = 3\cos\theta$

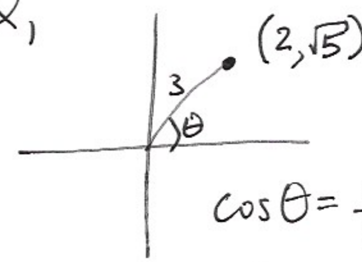
$\cos\theta = \frac{2}{3}$

$\theta = \cos^{-1}\frac{2}{3} \in [0, \pi] \longrightarrow \theta \in Q_1$

$r = 4 + \frac{2}{3} = \frac{14}{3}$

$x = r\cos\theta = \frac{14}{3} \cdot \frac{2}{3} = \frac{28}{9}$

$y = r\sin\theta = \frac{14}{3} \cdot \frac{\sqrt{5}}{3} = \frac{14\sqrt{5}}{9}$



$\cos\theta = \frac{x}{r}$

$r^2 = x^2 + y^2$

$3^2 = 2^2 + y^2$

$y = \pm\sqrt{5}$

$y = \sqrt{5}$ IN Q_1

BLUE AREA =

AREA INSIDE LIMAÇON W/ LOOP ON $[0, \cos^{-1}\frac{2}{3}]$

MINUS

" " CONVEX LIMAÇON ON $[0, \cos^{-1}\frac{2}{3}]$

LE AREA $= \int_0^{\cos^{-1}\frac{2}{3}} \frac{1}{2}(2+4\cos\theta)^2\,d\theta - \int_0^{\cos^{-1}\frac{2}{3}} \frac{1}{2}(4+\cos\theta)^2\,d\theta$

$$= \int_0^{\cos^{-1}\frac{2}{3}} \frac{1}{2}\left[(2+4\cos\theta)^2-(4+\cos\theta)^2\right]d\theta$$

$-1 = \cos 2\theta$

$= \frac{1}{2}(1+\cos 2\theta)$

$$= \frac{1}{2}\int_0^{\cos^{-1}\frac{2}{3}} \left(4+16\cos\theta+16\cos^2\theta-(16+8\cos\theta+\cos^2\theta)\right)d\theta$$

$(\theta+\frac{1}{2}\sin 2\theta)+C$

$$= \frac{1}{2}\int_0^{\cos^{-1}\frac{2}{3}} \left(-12+8\cos\theta+15\cos^2\theta\right)d\theta$$

ALL:

$= \frac{\sqrt{5}}{3}$

$= \cos^{-1}\frac{2}{3}$

$$= \frac{1}{2}\left(-12\theta+8\sin\theta+15\left(\frac{1}{2}\left(\theta+\frac{1}{2}\underbrace{\sin 2\theta}_{2\sin\theta\cos\theta}\right)\right)\right)\Bigg|_0^{\cos^{-1}\frac{2}{3}}$$

$$= \frac{1}{2}\left(-12\theta+8\sin\theta+\frac{15}{2}\theta+\frac{15}{2}\sin\theta\cos\theta\right)\Bigg|_0^{\cos^{-1}\frac{2}{3}}$$

$$= \frac{1}{2}\left(-\frac{9}{2}\theta+8\sin\theta+\frac{15}{2}\sin\theta\cos\theta\right)\Bigg|_0^{\cos^{-1}\frac{2}{3}}$$

(9)

$$= \frac{1}{2}\left(-\frac{9}{2}\cos^{-1}\frac{2}{3}+8\cdot\frac{\sqrt{5}}{3}+\frac{\cancel{15}^{\,5}}{\cancel{2}}\frac{\sqrt{5}}{\cancel{3}}\frac{\cancel{2}}{3}\right)$$

$f(\theta), \theta, \theta_1, \theta_2)$

$$= -\frac{9}{4}\cos^{-1}\frac{2}{3}+\frac{4}{3}\sqrt{5}+\frac{5}{6}\sqrt{5}$$

$f(\theta)\,d\theta$

$$= \frac{13}{6}\sqrt{5}-\frac{9}{4}\cos^{-1}\frac{2}{3}$$

$$\text{YELLOW} = \int_0^{\cos^{-1}\frac{2}{3}} \frac{1}{2}(4+\cos\theta)^2\,d\theta + \int_{\cos^{-1}\frac{2}{3}}^{\frac{2\pi}{3}} \frac{1}{2}(2+4\cos\theta)^2\,d\theta$$

$$- \int_{\pi}^{\frac{4\pi}{3}} \frac{1}{2}(2+4\cos\theta)^2\,d\theta$$

ARCLENGTH

PARAMETRIC

$$\begin{aligned} x &= f(t) \\ y &= g(t) \end{aligned} \quad t \in [a,b] \qquad \text{ARCLENGTH } S = \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2}\,dt$$

POLAR $r = f(\theta)$

$$\begin{aligned} x &= r\cos\theta = f(\theta)\cos\theta \\ y &= r\sin\theta = f(\theta)\sin\theta \end{aligned} \quad \theta \in [\theta_1, \theta_2]$$

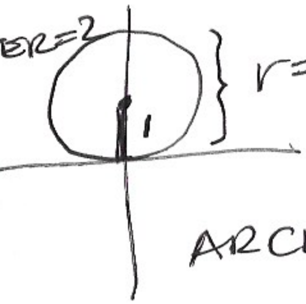
$$\text{ARCLENGTH} = \int_a^b \sqrt{(f'(\theta)\cos\theta - f(\theta)\sin\theta)^2 + (f'(\theta)\sin\theta + f(\theta)\cos\theta)^2}\,d\theta$$

$$\ldots\text{os}^2\theta - \cancel{2f'(\theta)f(\theta)\cos\theta\sin\theta} + (f(\theta))^2\sin^2\theta$$

$$\ldots\text{in}^2\theta + \cancel{2f'(\theta)f(\theta)\sin\theta\cos\theta} + (f(\theta))^2\cos^2\theta$$

$$\ldots))^2\overset{1}{\cancel{(\cos^2\theta+\sin^2\theta)}} + (f(\theta))^2\overset{1}{\cancel{(\sin^2\theta+\cos^2\theta)}}$$

$$= \int_a^b \sqrt{(f'(\theta))^2 + (f(\theta))^2}\,d\theta$$



$r = 2\sin\theta$

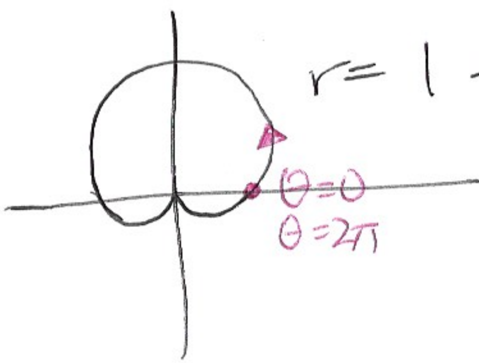
ARCLENGTH $= 2\pi$

$$\int_0^{\cancel{2\pi}\ \pi} \sqrt{((2\sin\theta)')^2 + (2\sin\theta)^2}\, d\theta$$

$$= \int_0^{\cancel{2\pi}\ \pi} \sqrt{4\cos^2\theta + 4\sin^2\theta}\, d\theta$$

$$= \int_0^{\cancel{2\pi}\ \pi} \sqrt{4}\, d\theta$$

$$= 2(\cancel{2}\pi - 0) = \cancel{4\pi}\ 2\pi$$



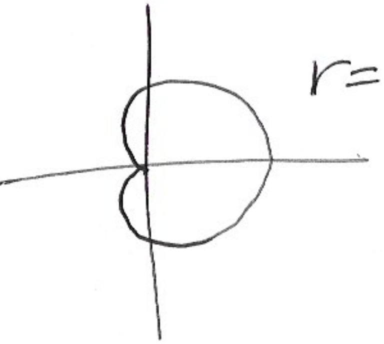
$r = 1 + \sin\theta$

$$\int_0^{2\pi} \sqrt{((1+\sin\theta)')^2 + (1+\sin\theta)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{2 + 2\sin\theta}\, d\theta = 8$$

BY CALCULATOR



$r = 1 + \cos\theta$

$$\int_0^{2\pi} \sqrt{(-\sin\theta)^2 + (1+\cos\theta)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{2 + 2\cos\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{4\cos^2\frac{\theta}{2}}\, d\theta$$

$$= \int_0^{2\pi} \left|2\cos\frac{\theta}{2}\right| d\theta$$

$$= \int_0^{\pi} 2\cos\frac{\theta}{2}\, d\theta + \int_{\pi}^{2\pi} \left(-2\cos\frac{\theta}{2}\right) d\theta$$

$$= 4\sin\frac{\theta}{2}\Big|_0^{\pi} - 4\sin\frac{\theta}{2}\Big|_{\pi}^{2\pi}$$

$$= 4(1-0) - 4(0-1)$$

$$= 8$$

$\cos 2\theta = 2\cos^2\theta - 1$

LET $\theta = \frac{\theta}{2}$

$\cos\theta = 2\cos^2\frac{\theta}{2} - 1$

$+\cos\theta = 2\cos^2\frac{\theta}{2}$

$+ 2\cos\theta = 4\cos^2\frac{\theta}{2}$

$\cos\frac{\theta}{2} > 0$

IF $\frac{\theta}{2}$ IN Q_1, Q_4

$\cos\frac{\theta}{2} < 0$

IF $\frac{\theta}{2} \in Q_2, Q_3$

$\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{2}$

$\pi < \theta < 3\pi$